### EE 435

Lecture 41

### Switched-Capacitor Circuits •SC Amplifiers •SC Filters

### Sample and Hold Circuits



 $v_n(mT)$  for each m is a random variable with some distribution function This distribution function is independent of m (i.e. the variables are identically distributed) Assume  $\mu_n$  is the mean and  $\sigma_n$  is the standard deviation of this random variable What is the relationship, if any, between  $v_n$  and  $\hat{v}_m$  **Theorem 1** If v(t) is a continuous-time zero-mean noise source and  $\langle v(kT) \rangle$  is a sampled version of v(t) sampled at times T, 2T, .... then the RMS value of the continuous-time waveform is the same as that of the sampled version of the waveform. This can be expressed as  $v_{\rm RMS} = \hat{v}_{\rm RMS}$ 

**Theorem 2** If v(t) is a continuous-time zero-mean noise signal and  $\langle v(kT) \rangle$  is a sampled version of v(t) sampled at times T, 2T, .... then the standard deviation of the random variable v(kT), denoted as  $\sigma_v$  satisfies the expression  $\sigma = v = \hat{v}$ 

From Theorem 1 we obtain the RMS value of the switched capacitor sampler

#### Review from Last Lecture Sample and Hold Circuits



RMS noise at output of basic SC S/H is independent of R but dependent upon C

#### Review from Last Lecture Some of the most basic and widely used analog circuits



But ratio accuracy can be very good (0.1% or better with good layout and appropriate area)



#### Both are orders of magnitude unacceptable !

#### Review from Last Lecture An amplifier alternative ?



**Inverting Amplifier** 

- Capacitor version is area effective and can have very good accuracy
- The node between C<sub>1</sub> and C<sub>2</sub> is a floating node if the Op Amp has a MOS differential pair at the input
- But if we get any charge on the intermediate node there is no way to get it off

#### **Review from Last Lecture**

### An amplifier alternative ?:





 $\Phi_1$  and  $\Phi_2$  are nonoverlapping clocks

#### During $\Phi_1$

 $C_1$  is charged to  $V_{IN}$  and stores charge  $Q_1 = C_1 V_{IN}$ 

 $C_F$  is discharged and  $V_{OUT}=0$ 

During  $\Phi_2$ 

 $C_1$  is discharged but charge is transferred to  $C_F$ 

 $Q_2$ =- $Q_1$  and  $V_{OUT}$ = $Q_2/C_F$ 

Substituting for Q<sub>1</sub> we obtain  $V_{OUT} = -\frac{C_1}{C_2}V_{IN}$ 

Serves as a voltage amplifier with output valid during  $\varphi_2$ 

#### **Review from Last Lecture**

### An amplifier alternative !



 $V_{OUT} = -\frac{C_1}{C_r}V_{IN}$ 



 $\Phi_1$  and  $\Phi_2$  are nonoverlapping clocks

- Many applications only need amplifier output at discrete points in time
- Accuracy can be very good
- Area can be very small

But, what about the switches?

**Review from Last Lecture** 

### Switches for SC Circuits



- Often a single MOS transistor is adequate (either n-ch or p-ch)
- Sometimes need transmission-gate switch (parallel n-ch and p-ch)
- Switches work very well and can be very small but must manage their R<sub>ON</sub>



And parasitic capacitors do not match very well

And parasitic capacitances may be highly nonlinear (junction capacitors)

### **Parasitic Capacitances**



10 parasitic capacitances!



 $C_1/C_F$  can be very precisely controlled with appropriate layout and area allocation

If op amp is ideal,  $C_{d1}$ ,  $C_{B1}$ ,  $C_{s2}$ ,  $C_{s3}$ ,  $C_{Bc}$ ,  $C_{Tc}$  and  $C_{d3}$  do not affect charge transfer !

But  $C_{s1}, C_{T1}, C_{d2}$  are all in parallel with  $C_1$  and all transfer charge

$$V_{OUT} = -\frac{C_1 + (C_{s1} + C_{T1} + C_{d2})}{C_r} V_{IN}$$

Parasitic capacitances not accurately controlled and dramatically degrade matching!

#### Stray Insensitive SC Amplifiers



Another SC Amplifier with even more switches !

Increased to 14 diffusion parasitic capacitances



Can show that all 14 diffusion parasitic capacitances do not affect gain !!

#### Stray Insensitive SC Amplifiers



Can show that all diffusion parasitic capacitances do not affect gain Gain can be accurately controlled !

Summing amplifier inputs either inverting or noninverting can be easily obtained



# Consider the $\mathop{\text{Basic}}_{\rm c}$ Integrator



Key performance of integrator (and integrator-based filters) is determined by the integrator time constant  ${\sf I}_0$ 

Precision of time constants of a filter invariably determined by precision of I<sub>0</sub>

### **Integrator-Based Filters:**



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_1} \frac{s}{s^2 + s \left(\frac{1}{R_0 C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

Denote as a two-integrator-loop structure

### **Integrator-Based Filters:**



### **Integrator-Based Filters:**



Second-order Lowpass Filter

Denote as a two-integrator-loop structure

- Any filter transfer function can be implemented with integrators and summers
- Some of the best known filter structures are based upon integrators and summers
- Accuracy of RC products is critical in the design of good filters

# **Consider the Basic Integrator**



Accurate control of  $I_0$  is required to build good filters !

- 1. Accuracy of R and C difficult to accurately control particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
- 2. Size of R and C unacceptably large if I<sub>0</sub> is in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

Appears to be Incredible Challenge to Building Filters on Silicon!

# **Integrator Design Issues**



Consider:



### **Consider the Switched-Capacitor Circuit**



### **Consider the Switched-Capacitor Circuit**



# Compare the performance of the following two circuits



Consider the charge transferred to the feedback capacitor for both circuits in an interval of length  $T_{CLK}$  at arbitrary time  $t_1$ 

For the RC circuit:



Since  $V_{in}$  changes slowly assume input is constant over one clock period

$$Q_{RC} \simeq \int_{t_1}^{t_1 + T_{CLK}} \frac{V_{in}(t_1)}{R} dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{t_1 + T_{CLK}} 1 dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{T_1 + T_{CLK}} 1 dt$$



Consider the charge transferred to the feedback capacitor for both circuits in an interval of length  $T_{\rm CLK}\,$  at time  $t_1$ 

For the RC circuit:

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Observe that a resistor "transfers" charge proportional to  $V_{\mbox{\scriptsize in}}$  in a short interval of  $T_{\mbox{\scriptsize CLK}}$ 

For the SC circuit





Since V<sub>in</sub>(t) is slowly varying

 $Q_{C1}\simeq C_{1}V_{in}\left(t_{1}\right)$ 

But this is the charge that will be transferred to C during phase  $\Phi_2$ 

 $Q_{SC} \simeq C_1 V_{in}(t_1)$ 

Observe that the SC circuit also transfers charge proportional to  $V_{\text{in}}$  in short intervals of length  $T_{\text{CLK}}$ 



Comparing the two circuits

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Equating charges since both proportional to  $V_{in}(t_1)$ 

$$C_{1} \approx \left[\frac{1}{R}\right] T_{CLK}$$
$$R_{EQ} \approx \frac{1}{f_{CLK}C_{1}}$$



Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

Note that large resistors require small capacitors !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!



- 1. Accuracy of R and C difficult to accurately control particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
- Size of R and C unacceptably large if I<sub>0</sub> is in audio frequency range (2 or 3 orders of magnitude too large)
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Appears to be Incredible Challenge to Building Filters on Silicon!

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

Equivalence Between Rapidly Switched Capacitor and Resistor



$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$



This is a frequency referenced filter!

Note that IOeq can be very accurately controlled !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!



### Recall:

- 1. Accuracy of R and C difficult to accurately control particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
  - Size of R and C unacceptably large if I<sub>0</sub> is in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

#### Appears to be Incredible Challenge to Building Filters on Silicon!

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

# Neither observation by itself was particularly useful but realizing both is really significant!

### The Observations about the SC integrator





- 1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
- 2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

- 1. Accuracy of cap ratio and  $f_{CLK}$  very good
- 2. Area of C1 and C not too large
- 3. Amplifier GB limits performance less

### The Genius !!



- 1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
- 2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
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- 1. Accuracy of cap ratio and f<sub>CLK</sub> very good
- 2. Area of C1 and C not too large
- 3. Amplifier GB limits performance less

Observation of Maxwell (and other "Me Too" up until 1977) on equivalence of resistor and switched capacitor had no impact

Two groups independently observed items 1) and 2) in 1976/1977 timeframe and realized that practical implementations on silicon were possible and that is the genius of the concept

Switched Capacitors and the corresponding charge redistribution circuits now used well beyond the SC filter field

Incredible enthusiasm and effort followed for better part of a decade

### sC integrator with summing inputs



### sC low-pass filter with summing inputs



### Consider again the SC integrator





Observe this circuit has considerable parasitics



 $C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1}$ 

Parasitic capacitors  $C_{s1}+C_{d2}+C_{T1}$  difficult to accurately match

- Parasitic capacitors of THIS SC integrator limits performance
- Other SC integrators (discussed next) ofter same benefits but are not affected by parasitic capacitors

#### Stray insensitive Inverting and Noninverting SC integrators



# Stray Insensitive SC Low-Pass Filter with Inverting and Noninverting Inputs

![](_page_41_Figure_1.jpeg)

Arbitrary number of inverting and ioninverting Inputs can be added

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}?$  For  $T_{CLK}{<<}T_{SIG}$ 

![](_page_42_Figure_2.jpeg)

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}$ ?

For T<sub>CLK</sub> <T<sub>SIG</sub>

![](_page_43_Figure_3.jpeg)

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}$ ?

For T<sub>CLK</sub><<T<sub>SIG</sub>

![](_page_44_Figure_3.jpeg)

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}$ ?

![](_page_45_Figure_2.jpeg)

![](_page_46_Figure_0.jpeg)

Considerable change in V(t) in clock period

![](_page_47_Figure_0.jpeg)

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}$ ?

![](_page_48_Figure_2.jpeg)

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$ 

for any  $T_{CLK}$ , characterized in time domain by difference equation

or in frequency domain characterized by transfer function obtained by taking z-transform of the difference equation

$$H(z) = -\frac{\frac{C_1}{z}}{\frac{z}{z-1}}$$

What is really required for building a filter that has high-performance features?

![](_page_49_Figure_1.jpeg)

Frequency domain:

Transfer function

 $T(s) = \frac{1}{RCs}$ 

Time domain:

**Differential Equation** 

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation

What is really required for building a filter that has high-performance features?

![](_page_50_Figure_1.jpeg)

Time domain:

**Differential Equation** 

Difference Equation

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$ 

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation

What if  $T_{CLK}$  is not much-much smaller than  $T_{SIG}$ ?

![](_page_51_Figure_2.jpeg)

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$ 

![](_page_51_Figure_4.jpeg)

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how  $T_{CLK}$  relates to  $T_{SIG}$ 

But good layout techniques and appropriate area need to be allocated to realize this potential !

#### Consider the following circuit

Termed a flip-around amplifier

![](_page_52_Figure_2.jpeg)

Clock signals are complimentary non-overlapping

### The flip-around amplifier $_{\Phi_2}$

![](_page_53_Figure_1.jpeg)

![](_page_53_Figure_2.jpeg)

![](_page_54_Figure_0.jpeg)

$$\mathbf{Q}_{1} = \mathbf{C}_{1} \left( \mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$
$$\mathbf{Q}_{2} = \mathbf{C}_{2} \left( \mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$

### The flip-around amplifier

![](_page_55_Figure_1.jpeg)

During  $\Phi_2$ 

![](_page_55_Figure_3.jpeg)

### The flip-around amplifier

![](_page_56_Figure_1.jpeg)

![](_page_56_Figure_2.jpeg)

$$Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{IN} - V^+) + C_1 (V_{IN} - V_X) = (C_1 + C_2) V_{IN} - C_2 V^+ - C_1 V_X$$

# The flip-around amplifier During $\Phi_2$

![](_page_57_Figure_1.jpeg)

 $Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{IN} - V^+) + C_1 (V_{IN} - V_X) = (C_1 + C_2) V_{IN} - C_2 V^+ - C_1 V_X$  $V_{C2F} = \frac{Q_{2F}}{C_2} = \left(1 + \frac{C_1}{C_2}\right) V_{IN} - V^+ - \frac{C_1}{C_2} V_X$  $V_{OUTF} = V_{C2F} + V^+ = \left(1 + \frac{C_1}{C_2}\right) V_{IN} - \frac{C_1}{C_2} V_X$ 

Comparison of Flip Around Amplifier with previous SC amplifier

![](_page_58_Figure_1.jpeg)

If  $V_X=0$ , both have a positive gain but somewhat more gain for a given capacitor ratio for the flip-around structure

In both cases, gain accuracy dependent upon how closely the capacitor ratios can be controlled

One particularly useful application is where want dc gain equal to 2 (1bit/stage pipeline ADC)

Flip-around requires matching two capacitors, other requires ratio matching of two capacitors

#### Another Flip Around Amplifier

![](_page_59_Figure_1.jpeg)

#### Clock signals are complimentary non-overlapping

#### Another Flip Around Amplifier

During phase  $\phi_1$ 

![](_page_60_Figure_2.jpeg)

Assume C<sub>B</sub> discharged at start of phase – must verify later

$$\begin{split} \boldsymbol{Q}_{\text{CA1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \\ \boldsymbol{Q}_{\text{CB1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \end{split}$$

$$V_{\text{OUT}} = -\frac{Q_{\text{CB1}}}{C_{\text{B}}} = -\frac{C_{\text{A}}}{C_{\text{B}}}V_{\text{IN}}$$

#### Another Flip Around Amplifier

During phase  $\phi_2$ 

![](_page_61_Figure_2.jpeg)

From phase  $\phi_1$   $\label{eq:QCA1} \begin{aligned} Q_{CA1} &= C_A V_{IN} \\ Q_{CB1} &= C_A V_{IN} \end{aligned}$ 

$$Q_{CA2} = Q_{CA1} + Q_{CB1}$$
$$Q_{CB2} = 0$$
$$V_{OUT} = -\frac{Q_{CA2}}{C_A}$$
$$V_{CB} = 0$$

Verified that  $C_B$  was discharged at the start of phase  $\phi_1$ 

$$V_{\text{OUT}} = -\frac{C_{\text{A}}V_{\text{IN}} + C_{\text{A}}V_{\text{IN}}}{C_{\text{A}}} = -2V_{\text{IN}}$$

This structure has a gain of 2 independent of any capacitor matching!

Can modify to get noninverting gain and gains of 3, 4, .., without matching requirements

### Non-overlapping Clocks

![](_page_62_Figure_1.jpeg)

- Essential that the clocks be non-overlapping
- Simple inverter to derive the complimentary clock will not work
- Must guarantee non-overlap in the presence of PVT variations
- In non-demanding speed applications,  $\phi_1$  and  $\phi_2$  will have 25% duty cycles

![](_page_63_Picture_0.jpeg)

# Stay Safe and Stay Healthy !

### End of Lecture 41